# KAZAK NATIONAL UNIVERSITY

## after aI Farabi

Mechanics and Mathematics Faculty Department of mathematics

## It is confirmed

at session of the faculty's Scientfic Council Protocol № from « » 2022

Faculty's Dean

**Syllabus**

**Actual Problems of Mathematics**

1 course, Spring semester 2023-2024

## Lector's name (Lecture and practical studies): Kaisar Tulenbaev

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* **Purpose:** to train of students with the basic methods of Model Theory
* **Problems:** the students are expected to be able to:

1. Manipulate propositions, sets, first-order formulae, relations, graphs;
2. Solve transitive, reflexive and symmetric closure of relations;
3. Doing basic problems on Model Theory;
4. Constructing propositions and formulae expressing given sentences;
5. Understand applications of graphs and trees;
6. Use Method of Quantify Elimination.

## Results of training

* General competences: too/. ability to study throughout life as a basis of continuous training in a context both personal professional, and social life

*interpersonal.* it is ready to work in collective; it is capable to find organizational and administrative solution in non-standard situations and it is ready to bear for them responsibility; is able to estimate critically the merits and demerits, to plan ways and to choose development tools of advantages and elimination of shortcomings; realizes the social importance of the future profession, possesses high motivation to performance of professional activity; it is capable to analyze social and significant problems and processes; *system:* to make non-standard decisions, to be able to cooperate with other people, reasonably to accept or reject someone's actions

* Content: The course is devoted to the basic questions of Model Theory. the concepts of a first- order language (formulas of predicate logic), the concept of structure, truth of a formula (sentence) on a structure, the concept of elementary theory of structure, models of elementary theory are introduced. elementary submodel, model completeness. The main theorems of the course are the Tarski-Vaught theorems on an elementary submodel, Tarski's criterion for the elimination of quantifiers of a complete theory, and the Fraisse-Ehrenfeucht-Taimanov criterion on the elementary equivalence of two structures. The course provides examples of complete theories Applied for computer systems of processing of information and management.
* **Prerequisites:** Discrete Mathematics and Mathematical Logic, Symbolic (Mathematical) Logic.
* **Post-requisites:** Model Theory and Algebraic Geometry.

# STRUCTURE AND CONTENT OF DISCIPLINE

|  |  |  |  |
| --- | --- | --- | --- |
| Week | **Specialties «Mathematics», 2 credits** | | |
| **Themes** | hour | Maximal points |
|  |  |  |  |
|  | Lecture 1 Language of Logic of Propositions. Elementary  propositions. Logical Operations. Truth Table for essential logical propositions. Examples. Propositional Equivalences. Table of Logical Equivalences. Definition of tautology and contradiction. | 1 | 3 |
| **Practical studies 1.** Prove that every formula of Logic of Propositions is logically equivalent to formula in disjunctive normal form (DNF) and in conjunctive normal form (CNF). | 1 |  |
| **IWST** 1 Number of non-equivalent formulas of Logic of |  |  |
| Propositions of n elementary propositions. |  |  |
| 2 | **Lecture** 2 Let A be a countable set. Then the set W(A) of all  finite words in Alphabet A is countable set. Corollary. | 1 | 4.5 |
|  | **Practical studies 2.** The set of all finite subsets of a set of | 1 |  |
|  | natural numbers is countable set. |  |  |
|  | IWST 2. The set of all algebraic numbers is a countable set. |  |  |
| 3 |  |  | 4.5 |
|  | Lecture 3 Language of First Order Logic. Alphabet. Terms.  Formulas. Sentences. Examples with explanations.  **Practical studies** 3 Definition of structure of signature Sigma. Examples with explanation. Definition of hold (truth) of sentence in structure (formula and structure of the same signature)..  **IWST** 3 Definable sets — definition. Examples with explanation. . Examples of structure, substructure. A subset of a structure that is a structure but not a substructure | 1  1 |  |
| 4 | **Lecture** 4 Structures with one 2-p1ace predicate. Axioms of  relation of equivalence. Axioms of relation of linear order.  **Practical studies** 4 . Equivalence classes. Its properties. Partition.  IWST 4. Intersection and union of equivalence relations | 1  1 | 4.5+  15 (quiz) |
| 5 | **Lecture** 5 Properties of binary relations  **Practical studies** 5 Deciding whether or not a given relation is reflexive, symmetric, transitive, etc.  **IWST 5.** Investigating relations between asymmetry and antisymmetry with reflexivity. | 2  1 | 4.5+  15 (IWS) |

|  |  |  |  |
| --- | --- | --- | --- |
| 6 | **Lecture** 6 Definition of isomorphism of two structures of the same signature. Examples with explanations. Theorem.  Number of non-isomorphic structures of cardinality m of finite relational signature is finite number. Evaluate.  **Practical** studies 6 Two countable dense linear order without ends are isomorphic.  IWST 6 The number of non-isomorphic structures of cardinality 3 with two 2-p1ace predicate. | i  1 | 4.5+  15 (quiz) |
| 7 | Lecture 7 Elimination of quantifiers. Test of Tarskii.  Negation of quantify free formula is quantify free formula. Theorem on necessary and sufficient condition to have the QE (quantifiers elimination) for the structure.  **Practical studies** 7 Any formula of Logic of Predicates is equivalent to formula in Prenex Normal Forma (PNF) **IWST 7.** Elimination of quantifiers of M= <N; = >, N is an infinite set. | 1  1 | 4.5+  25 (colloquium) |
|  | **Midterm** |  | **100** |
|  |  |  |  |
| 8 | **Lecture** 8 Dense linear order.  Elimination of quantifiers (M=<Q; =, < ), Q is the set of all rational numbers.  **Practical studies** 8 Definable sets in the structure. Operations on the family of definable sets. Examples.  **.IWST 8.** Write the following formula in the prenex normal form: (Hx3y (H (x) A K (y, x))) —+ 3xHz (K (x, z) —› H (y)). | 1  1 | 4.5 |
| 9 | **Lecture** 9 Algorithm of Euclid in K[x], in the ring of  polynomials over field K.  **Practical studies** 9 Modified Algorithm of Euclid in A[x], in the ring of polynomials over ring A.  **IWST 9.** Number of roots of polynomial f(x) from K[x]. | 1  1 | 4.5 |
| **10** | **Lecture 10** Real closed field (M= <R; <, +,-,•, 0,1 >).  Theorem of Tarskiy-Zeidenberg as generalization of Viet theorem.  **Practical studies 10** Axioms and semi-algebraic sets. IWST 10. Table of finite system of polynomials with real coefficients. | 1  1 | 4.5 |
| **11** | **Lecture 11.** Algebraic closure of set A in structure M. | 1 | 4.5 |
|  | Definition and three properties of algebraic closure. |  |  |
|  | **.Practical studies 11** Investigating particular cases. IWST 11. Graph representation of binary relations. | 1 |  |
| **12** | **Lecture 12** Definition of elementary submodel. Test of  Tarskii on elementary submodel. Definition of model complete theory.  **Practical studies 12** Theorem. If theory T admits QE then T is model complete.  **IWST 12.** Theorem. The set of Algebraic numbers is algebraic closed field (ACF). | 1  1 | 4.5+  15 (quiz) |

|  |  |  |  |
| --- | --- | --- | --- |
| 13 | Lecture 13 Definition of elementary equivalence of two structures. Definition of Game of Erhenfoiht.  **Practical studies 13** Wining strategy of first Garner in Game on the structure (M=<Q; =, < ), Q is the set of all rational numbers.  IWST 13. An elementary submodel. An example of a substructure that is not an elementary submodel. An example of a subset that is not a substructure. | 1  i | 4.5+  9 (IWS) |
| 14 | Lecture 14 Theorem of Fraisse —Taimanov -Ehrenfeuht  **Practical studies 14** Deciding whether or not a given graph is planar.  IWST 14. Proof of Kuratowski's theorem. | 1  1 | 4.5+  15 (quiz) |
| 15 | Lecture 15 Definition of a model complete theory.  **Practical studies 15** An example of a model complete theory.  IWST 15 An example is not a model complete theory. | 1  1 | 4.5+  25 (colloquium) |
|  | End term |  | **100** |
|  | **Final exam** |  | **100** |
|  | **Total** |  | **100** |

Table of estimation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| № |  | Maximum  points | Minimum points | Remark |
| 1 | Midterm | 100 | 50 | Sum of points of  weeks 1-7 |
| 2 | End term | 100 | 50 | Sum of points of  weeks 8-15 |
| 3 | Total term=  (midterm + end term)/2 | 100 | 50 | An average  arithmetic sum of midterm and  end term |
| 4 | Final exam | 100 | 50 |  |
| **5** | **Total** | **100** | 50 | An average  arithmetic sum  **of total term**  and final exam |

# LITERATURE

## Basic:

1. Н. К. Верещагин, А. Шень. Языки и исчисления. Москва, 2000.
2. Wilfrid Hodges. A shorter model theory. Cambridge University Press. 2000.
3. Kennet Rosen. Discrete Mathematics and Its Applications. William C Brown Pub. 824 pages. 1998.

## Additional:

* 1. С.В. Яблонский, Дискретная математика
  2. A. Mendelson. Mathematical Logic.

POLICY OF THE ACADEMIC BEHAVIOR AND ETHICS

The given program of a course regulates studies in the form of lecture both laboratory researches according to the schedule and the given program.

Tasks for independent work are given out by the tutor of lecture studies, reception of independent work is carried out also by the tutor of lecture studies in target dates, it accepts boundary tasks.

By the confirmed rules the student can pass no more than three studies. If the student has passed more than three studies (for the disrespectful reasons) the tutor is obliged to inform the student that it is deducted from the given training course.

One delay on study and-or leaving before the termination of study for any reasons will be considered as one passed study which is not subject to restoration.

Cellular telephones should be switched off in an audience. Non-observance of this rule is infringement of requirements.

Be tolerant, respect another's opinion. Objection formulate in the correct form. Plagiarism and other forms of dishonest work are inadmissible. Are inadmissible prompting and writing off during delivery IWS, the intermediate control and final examination, copying of the solved problems by other persons, passing an examination for other student. The student convicted of falsification of any information of a course, unapproved access to the Intranet, using crib, will receive a total estimation «F».

Help: Behind consultations on performance of independent works (IWS), their delivery and protection, and also for the additional information on the passed material and all other arising questions at a readable course address to the teacher in its offices-hours.

## The forms of knowledge's control

* Control works: 4 works
* Colloquium: 2 times
* Final exam: during examination session

The scale of knowledge's estimation

|  |  |  |  |
| --- | --- | --- | --- |
| Alphabetic estimation | Digital estimation (GPA) | Points (%) | Traditional estimation |
| A | 4 | 95-100 | “Excellent” |
| A- | 3,67 | 90-94 |
| B+ | 3,33 | 85-89 | “Good” |
| B | 3 | 80-84 |
| B- | 2,67 | 75-79 |
| C+ | 2,33 | 70-74 | “Satisfactorily” |
| c | 2 | 65-69 |
| c- | 1,67 | 60-64 |
| D+ | 1,33 | 55-59 |
| D | 1 | 50-54 |
| F |  | 0-49 | “Unsatisfactorily” |
| I |  |  | “The discipline isn't over” |
| W |  |  | “Giving the discipline up” |
| AW |  |  | “is sending down from the  discipline” |
| AU |  |  | “The discipline is listened” |
| P/NP(Pass / No Pass) |  | 65-100/0-64 | " The discipline is passed/ The discipline is not passed" |

*It is considered on chair meeting*

*the report Ne from « » 2024.*

Chair's chief

Lector K.Tulenbaev